AGENA ROCKET NOZZLE:
PROPERTIES AND GEOMETRY

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ABSTRACT

A Bell Aerospace Agena rocket nozzle is analyzed using simplified one dimensional isentropic ideal gas flow equations. Formulas for ratios of temperature, pressure and area of one dimensional isentropic nozzles are reviewed. A computer program for determining the ratios of temperatures, pressures and areas of a nozzle is developed. The program calculates nozzle fluid properties when given pressure data. An Agena rocket nozzle is analyzed using data calculated from the computer program. Nozzle geometry and fluid properties are tabulated and plotted. Results are discussed.

INTRODUCTION

The Bell Aerospace Agena has earned a reputation as a simple and reliable rocket engine. Rocket engines create thrust by accelerating a fluid through a nozzle. Converging-Diverging nozzles combined with the correct pressures can accelerate a fluid to supersonic velocities. The magnitudes of velocities, pressures and temperatures cause precise modeling to become complicated quickly due to friction, boundary layers and non-constant fluid composition. Assumption of one dimensional isentropic conditions allows for simple relationships between the fluid properties of the nozzle.

Nozzle geometry and fluid properties along the length of the Agena nozzle are to be found. The known conditions of the rocket nozzle are,

$$\dot{m} = 1000 \text{ lbm/s} \quad \mathrm{p_{tc}} = 3000 \text{ psia} \quad \mathrm{T_{tc}} = 3700\ ^\circ\mathrm{R} \quad \mathrm{p_e} = 1.07 \text{ psia}$$

Where $\dot{m}$ is the flowrate, p_{tc} is the combustion chamber pressure, T_{tc} is the combustion chamber temperature and p_e is the exit plane pressure. A pressure field is given by the following pressure functions.

$$\begin{aligned}
p(x) &= -1.174\ x^3 + 2.81\ x^2 - 44.57\ x + 2919 && 0 < x \le 10^{in} \\
p(x) &= 4.172E4\ \exp(-0.3352\ x) && 10^{in} < x \le 20^{in} \\
p(x) &= 5.76E6\ x^{-3.94} && 20^{in} < x \le 30^{in} \\
p(x) &= 3.6E5\ x^{-3.13} && 30^{in} < x \le 50^{in} \\
p(x) &= 6.79E4\ x^{-2.7} && 50^{in} < x \le 60^{in}
\end{aligned}$$

Ideal gas and isentropic equations will be used to develop governing relations for the properties of a fluid. The specific heat ratio, k, will be assumed to be constant. The objective is to determine the fluid properties and geometry of a supersonic rocket nozzle. A computer program will be developed and used to calculate pressure, temperature, Mach number, density, velocity, area and radius along the length of a rocket nozzle. Data will be tabulated and plotted. Results and implications of the analysis will be discussed. A sample calculation will be worked at the nozzle throat.

THEORY

The flow of fluid in a nozzle can be modeled by assuming ideal gas, isentropic flow and constant specific heat. The ideal gas assumption relates density, pressure and temperature.

$$p = \rho R T$$

Isentropic flow, no change in entropy, is possible if no heat transfer or friction occurs. The properties k ,ratio of specific heats, and the gas constant, R, fully specify an ideal gas.

Mach number is defined as the ratio of the local velocity to the sonic velocity. Sonic velocity of an ideal gas is a function of gas properties and temperature (Bathie). Thus,

$$M = \frac{V}{c} = \frac{V}{\sqrt{kRT}}$$

The stagnation properties of a flow are those properties which would result if the flow were isentropically brought to rest. Stagnation properties are constant in an isentropic flow. Thus, properties along the nozzle are best referenced against the stagnation properties.

With these assumptions of ideal gas and isentropic flow, ratios of pressure, density and temperature can be related to the stagnation pressure, density and temperature at a given Mach number (Handout).

$$\frac{P_0}{P} = \left(1 + \frac{k-1}{2} M^2\right)^{\frac{k}{k-1}}$$
$$\frac{T_0}{T} = 1 + \frac{k-1}{2} M^2$$
$$\frac{\rho_0}{\rho} = \left(1 + \frac{k-1}{2} M^2\right)^{\frac{1}{k-1}}$$

Solving for M in terms of $\frac{P_0}{P}$ yields,

$$M = \sqrt{\left(\frac{P_0}{P}^{\frac{k-1}{k}} - 1\right)\frac{2}{k-1}}$$

Additionally, the ratio of the local Area to the throat area can be specified by the Mach number (Handout).

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{2}{k+1}\left(1 + \frac{k-1}{2} M^2\right)\right]^{\frac{k+1}{2(k-1)}}$$

Where A^* is the throat area and A is the local nozzle area.

The mass flow rate as a function of nozzle geometry and fluid properties can be found from basic continuity (Bathie) where V is the average velocity, A is the nozzle area, ρ is the density and $\dot{m}$ is flowrate

$$\dot{m} = \rho \; V \; A$$

Substituting $M\sqrt{k \; R \; T}$ for V and solving for the Area yields,

$$A = \frac{\dot{m}}{\rho \; M\sqrt{k \; R \; T}}$$

Thus with a circular cross section, the radius is,

$$r = \sqrt{\frac{A}{\pi}}$$

Assuming ideal gas and isentropic allows for the calculation of properties along the nozzle. An example calculation of properties are made in Appendix D.

METHOD OF CALCULATION

A FORTRAN program, given in Appendix A, was developed to calculate pressure, temperature, Mach number, velocity, density, area and nozzle radius when given a pressure function and a mass flow rate. The program is divided into 3 steps; initialization, finding reference conditions and calculating conditions along the nozzle's length.

The program initialization consists of creating functions and variables, specifying data output format and setting the properties of the working fluid. Variables and functions are defined as single precision real. Variable usage is defined below.

- `K` ≡ `Ratio of specific heats`
- `R` ≡ `Gas Constant`
- `X` ≡ `Location along nozzle in inches`
- `PT` ≡ `Stagnation pressure` P_t
- `TT` ≡ `Stagnation temperature` T_t
- `RHOT` ≡ `Stagnation density,` ρ_t
- `MDOT` ≡ `Mass flow rate`
- `P` ≡ `Local pressure`
- `T` ≡ `Local temperature`
- `RHO` ≡ `Local density`
- `M` ≡ `Local Mach number`
- `V` ≡ `Local velocity`
- `RPLUS` ≡ `Nozzle radius (positive)`
- `RMINUS` ≡ `Nozzle radius (negative)`
- `MACH` ≡ `Function for Local Mach number given` P `and` P_t
- `TTT` ≡ `Function for` T/T_t `given Mach number`
- `PPT` ≡ `Function for` P/P_t `given Mach number`
- `RRT` ≡ `Function for` ρ/ρ_t `given Mach number`
- `PRESS` ≡ `Function for Pressure given X location`

The function PRESS is set up as a subroutine. An IF statement is used to select the pressure function from the X location supplied. The subroutine returns the local pressure at the specified x location.

Next in the initialization, an output file, ag.out, is opened. Calculated values will be sent to this file during program execution. The format statement, 1001, specifies the spacing and width of the output values. Next, a header line writes to the output file names and units of the data values.

Finally, fluid properties and conditions are initialized. Gas constant,R, and the specific heats ratio, k, of the working fluid are set. Stagnation properties P_t and T_t are set. Stagnation density is calculated from the stagnation pressure and temperature from theory.

The second division of the program is finding useful reference conditions. From the required flowrate, the nozzle's throat area is calculated. Using a known local and stagnation pressure, the program finds the Mach number at an x location along the nozzle. The resulting local temperature, density and area are calculated from theory at this x location. From knowledge of the area and Mach number, the area ratio (A/A^*) is used to find the throat area (A^*).

The final division of the program finds the conditions along the nozzle. A series of four DO loops are set up to step along the x distance while adapting the resolution of the nozzle in the areas of rapid changes. Each DO loop consists of the following. First, the `PRESS` function is used to find the local pressure. This local pressure is combined with the stagnation pressure to find the Mach number. Next, the local temperatures and densities are calculated from the Mach number. The local fluid velocities and area ratios are found from the temperature and density. The nozzle radii are calculated. Values of pressure, temperature, Mach number, velocity, density, area and radii are written to the output file for that particular x distance. Finally, after going through each DO loop, the program closes the output file and exits.

RESULTS AND DISCUSSION

Calculations were performed as described above and are tabulated in Appendix B. Plots of geometry and properties are given in Appendix C. An example calculation of properties at the nozzle throat is given in Appendix D.

The contour of the Agena nozzle is plotted in Figure 7 as radius versus x location. Overall, the nozzle looks reasonable and similar in shape to other common rocket nozzles. As expected, the nozzle is a converging-diverging nozzle intended to accelerate the fluid up to and then beyond Mach one. The throat radius is approximately 3.5 inches. The exit plane radius is 31 inches. In the aft part of the nozzle, the radius is approximately linear with the x distance.

A plot of pressure versus distance is given in Figure 1. As given in the pressure field function, the pressure drops from almost 3000 psi at the nozzle inlet to 1.07 psi (Table 1, Appendix B) at the nozzle exit. Approximately half of the pressure drop occurred in accelerating the fluid to Mach 1 at the throat. Aft of the throat, the pressure drops asymptotically towards zero pressure.

Temperature versus distance is plotted in Figure 2. As with pressure, the temperature drops from a high of 3600 Rankine at the inlet to 380 Rankine at the exit (Table 1). The temperature dropped less than a quarter of its stagnation temperature to accelerate the fluid to Mach 1. Most of the temperature drop occurred accelerating the fluid from Mach 1 to Mach 3 (Table 1). Changes between the pressure functions are difficult to see in the temperature plot.

A plot of Mach number versus distance is given in Figure 3. Mach number changed most rapidly near the throat, where the Mach number is required to be Mach one. Changes in slope and curvature in the Mach plot are easily identified as the changes in the pressure functions.

A Velocity versus distance plot is given in Figure 4. As expected, the velocity plot resembles the Mach number plot. The velocity plot has slightly less noticeable changes in curvature at the pressure function change points. Unlike the Mach number, the velocity plot flattens out in the aft portion of the nozzle. This can be explained by noticing that since the pressure in the aft portion doesn't change rapidly, the available energy to accelerate the flow is small. The change in Mach number is mainly due to the change in temperature, which is changing to accommodate the increasing area.

The plot for density versus distance is given in Figure 5. The density plot resembles the pressure plot. The maximum density is at the inlet and the minimum density is at the exit. Changes between pressure functions are not noticeable.

A plot of area versus distance is given in Figure 6. This plot clearly shows how area is related to the square of the radius. In the aft part of the nozzle, the area curve resembles a parabola. This plot is continuous without any visible sudden changes in curvature.

Overall, along the nozzle, the pressure, temperature and density decreased from inlet to outlet. The Mach number and velocity increased from inlet to exit. The area was first dropped to accelerate the flow to Mach one and then increased to accelerate the flow beyond Mach one.

The design of the program allowed for ease in adapting the step size in critical areas. As seen in the nozzle contour plot (Figure 7), the step size was reduced near the throat. For more complicated situations, this method would allow for easy "remeshing" of the steps to account for large gradients. In this problem, the change in step size increased the accuracy of the solution because of the rapid changes in M, P, T and ρ in the vicinity of the throat. Additionally, the use of FORTRAN allowed for ease in changing the number of significant digits through the manual manipulation of the FORMAT statement.

The pressure field caused problems due to the piecewise functions used to describe the field. The pressures at x=10 differed between the pressure functions for $0 < x \leq 10$ and $10 < x \leq 20$. For the first pressure equation, the pressure at x=10 was 1580.3 psi while the second pressure function gave 1460 psi. This discontinuity can be seen in the Mach number versus distance plot (Fig 3.) as a downward jump in the Mach number at x=10. Between the other pressure functions, the change was more continuous and only the curvature difference was noticeable in the Mach number plot.

The most severe problem in this analysis was the assumption of ideal air as a working fluid. The temperatures of the fluid, $T_t = 3700°R$ would render the assumption of constant

C_p invalid. Also, dissociation was not taken into account. In any case, the actual Agena engine used a hydrazine variant and nitric acid, not air. The assumption of ideal air does, however, allow for an easy and quick demonstration of accelerating air in a nozzle.

CONCLUSIONS

An Agena rocket nozzle was analyzed using a simplified fluid and flow model. The assumption of an ideal gas on an isentropic path led to simple relationships between the properties of a fluid in a nozzle and the nozzle geometry. A computer program was developed to calculate the properties along a nozzle when given the pressure field. The resulting Agena nozzle contour (Figure 7, Appendix C) is a converging-diverging nozzle. The nozzle was able to accelerate the air to Mach 6.6 or 6300 ft/s at 383 $^\circ R$ from a combustion pressure and temperature of 3000 psi and 3700 $^\circ R$.

The limitations of this analysis concern the fluid model used. A more precise method would be to use the proper Agena combustion products with a variable C_p and temperature dependent mixture. This method would greatly complicated the analysis and simulation requirements. For the purposes of investigating the properties of converging-diverging nozzles, the current method of ideal air and isentropic flow is sufficient.

REFERENCES

Bathie, William W. (1996)

Fundamentals of Gas Turbines, John Wiley & Sons, Inc.

MAE 4243 Class Handout (2000)

Isentropic Flow Equations.

APPENDIX A:

Computer Program

APPENDIX B:

Tabulated Results

APPENDIX C:

Figures

APPENDIX D:

Example Calculation at x=10 in

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At the nozzle throat, x=10, an example calculation yields the following.

Fluid properties:

$$p_{tc} = 3000 \text{ psia}$$

$$T_{tc} = 3700 \text{ °R}$$

Thus, the stagnation density is

$$\rho_t = P_t/RT_t = \frac{3000\ 144}{53.34370032.2} = 0.06798 \text{slug/ft}^3$$

At x=10[in], the pressure is $p(10) = -1.174\ 10^3 + 2.81\ 10^2 - 44.57\ 10 + 2919 = 1580.3[psi]$.

Calculating the Mach number from P_t and P yields,

$$M = \sqrt{\left(\frac{P_0}{P}^{\frac{k-1}{k}} - 1\right)\frac{2}{k-1}} = \sqrt{\left(\frac{3000}{1580.3}^{\frac{1.4-1}{1.4}} - 1\right)\frac{2}{1.4-1}} = 1.00$$

The local density is,

$$\rho = \frac{\rho}{\rho_t}_{M=1.0} \rho_t = \left(1 + \frac{1.4-1}{2} 1.00^2\right)^{\frac{1}{1.4-1}} 0.06798 = 0.04301 \text{slug/ft}^3$$

The local temperature is,

$$T = \frac{T}{T_0}\ T_0 = \frac{T}{T_0} = \frac{1}{1 + \frac{1.4-1}{2} 1.00^2}\ 3700 = 3080.8[R]$$

From the developed theory,

$$A = \frac{\dot{m}}{\rho\ M \sqrt{k\ R\ T}} = \frac{1000}{0.04301\ 1.00 \sqrt{1.4\ 53.3\ 3080.8\ 32.2}}\ \frac{144}{32.2} = 38.1[in^2]$$

Also the throat area is,

$$A^* = A/A^*\ A = \frac{1}{1.00}\left[\frac{2}{1.4+1}\left(1 + \frac{1.4-1}{2} 1.00^2\right)\right]^{\frac{1.00+1}{2(1.4-1)}} 38.1 = 38.1[in^2]$$

The radius is,

$$r = \pm\sqrt{A/\pi} = \mp\sqrt{38.1/3.1415} = \pm 3.48[in]$$