JET ENGINE PERFORMANCE

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ABSTRACT

A formula for thrust using only the properties external to the engine was derived. A computer program was made to calculate thrust for a range of inlet and exit velocities. The derived thrust formula was used to calculate the maximum thrust, power and efficiency of a jet engine. The results show that the ratio of inlet to exit velocities to obtain the maximum thrust differs from the ratio for maximum power or efficiency. Implications of these differences are discussed with respect to the selection of engines in jet aircraft.

INTRODUCTION

Jet engines are used to provide power for high performance applications. This power is developed by accelerating air through the engine to provide a net imbalance of force called thrust. The acceleration of air is provided by a series of rotating fans to compress the air and by the burning of a fuel to rotate the fans. Analyzing the performance by considering the individual part is immensely complex. A simpler approach considers the jet engine as a device to accelerate air from an inlet velocity to an exit velocity. Using a surface containing the entire jet and only considering only the inlet and exit properties crossing the surface simplifies analysis of performance.

The objective is to use a control surface to calculate the thrust and power of a jet engine within a range of inlet and exit velocities. The properties needed are intake velocity, exit velocity, density of fluid and area of intake. A computer program will use the derived formula for thrust to develop a table of thrust values over the inlet and exit velocity ranges. Thrust, power and efficiency will be derived and graphed over a useful range of inlet and exit conditions.

THEORY

A jet engine provides thrust by accelerating a fluid. Newton's second law gives that force equals mass times acceleration ($F = ma$). Since acceleration is the derivative of velocity, force (ma) is the time derivative of the momentum (mv). The Material Derivative, which gives a rate of change that is time and location specific with a dummy variable $\Phi = m\phi$, is

$$\frac{D\Phi}{Dt} = \frac{\partial\Phi}{\partial t} + u\frac{\partial\Phi}{\partial x} + v\frac{\partial\Phi}{\partial y} + w\frac{\partial\Phi}{\partial z} = \frac{\partial\Phi}{\partial t} + \iint_{cs} u\phi\,\vec{V}\cdot\hat{n}\,dA$$

From the Material Derivative with steady state conditions,

$$F = \frac{d(mv)}{dt} = \iint_{cs} u\rho\,\vec{V}\cdot\hat{n}\,dA$$

Control surface integration over input and output areas yields

$$\Sigma F = (-u_1\rho_1u_1A_1) + (u_2\rho_2u_2A_2)$$

The continuity equation of mass, $\dot{m}_{in} = \dot{m}_{out}$, requires $A_2 = (\rho_1 u_1 A_1)/(\rho_2 u_2)$, thus the force required for the acceleration of air from u_1 to u_2 is,

$$F = -\rho_1 u_1^2 A_1 + \rho_1 u_1 u_2 A_1 \ = \dot{m}(u_2 - u_1)$$

where $\dot{m} = \rho u_1 A$ is the mass flow rate through the control surface, u_1 is the fluid velocity at the intake and u_2 is the exit velocity of the fluid.

METHOD OF CALCULATION

A computer program written in the C language and given in Appendix A was created to output values for thrust given the flight inlet and exit velocities. The Variables are globally declared as floating point numbers. The Program first outputs the descriptions of variables and a horizontal index of exit velocities. Then two nested loops calculate and output values for the thrust along the range of inlet and exit velocities. The first loop, which varies inlet velocity, calculates and stores to a variable the mass flow rate $\dot{m}$. Within this loop, the inlet velocity is vertically outputted along with the calculated mass flow rate. While still inside the first loop, the second loop outputs the calculated thrust ($Thrust = \dot{m}(u_2 - u_1)$) if the thrust is positive ($u_2 > u_1$) for all exit velocities with the current inlet velocity. After first loop reaches and outputs the final inlet velocity, the program exits.

RESULTS AND DISCUSSION

Calculations have been performed as described above. A table of the resulting data with the inlet velocities ranging from 0 to 800 m/s and exit velocities also ranging from 0 to 800 m/s is given in in Appendix B (Tables). A graph of the data is given in Figure 1(Appendix C). With respect to inlet and exit velocity, the thrust of an jet engine takes the shape of an inverted parabola with zero thrust when at zero velocity and also when the inlet velocity equals the exit velocity. It was noticed that the maximum thrust occurs with an exit velocity twice that of the inlet velocity (See Figure 1). This can be inferred from the derivative of the expression for thrust set to zero

$$\frac{d(F)}{du_1} = -2\rho_1 u_1 A_1 + \rho_1 u_2 A_1 = 0$$

Therefore, maximum force is obtained at

$$u_2 = 2u_1$$

Power, force times velocity $(F \cdot u_1)$, is maximized at

$$\frac{d(Power)}{du_1} = 2\rho_1 u_1 u_2 A_1 - 3\rho_1 v_1^2 A_1 = 0$$

Therefore, maximum power occurs when

$$u_2 = 1.5u_1$$

The propulsive efficiency of the engine is (Cumpsty, 1997)

$$\eta_p = 2 \cdot \frac{u_1}{u_1 + u_2}$$

Therefore, the maximum efficency is at $u_1 = u_2$ (Figure 3).

CONCLUSION

From the derived thrust equation, it was found that,

1. If the force is to be maximized, the exit velocity relative to the vehicle must be twice the inlet velocity. (Figure 1.)
2. If the power is to be maximized, the exit velocity must be one and a half times the inlet velocity. (Figure 2.)
3. Maximum propulsive efficiency occurs when the inlet velocity is equal to the exit velocity. (Figure 3.)

The result of the maximum thrust, power and efficency occurring at different points makes jet engine parameters and design dependent upon the expected function of the engine. The internal characteristics of the jet engine control the relationship between inlet and exit velocities. When the lowest cost is required such as in a commercial transport jet, the engine selected needs to have an exit velocity near the inlet velocity for maximum efficiency and lowest fuel burn cost. In comparison, a fighter jet needs to have an engine which is required to provide maximum power regardless of the loss in efficiency. Thus, jet engines must be selected with the operating functions and expected parameters known.

REFERENCES

Cumpsty, Nicholas (1997)

Jet Propulsion, Cambridge University Press.

APPENDIX A

Computer Program

APPENDIX B

Tables

APPENDIX C

Figures

Figure 1. Thrust per unit area versus exit velocity.

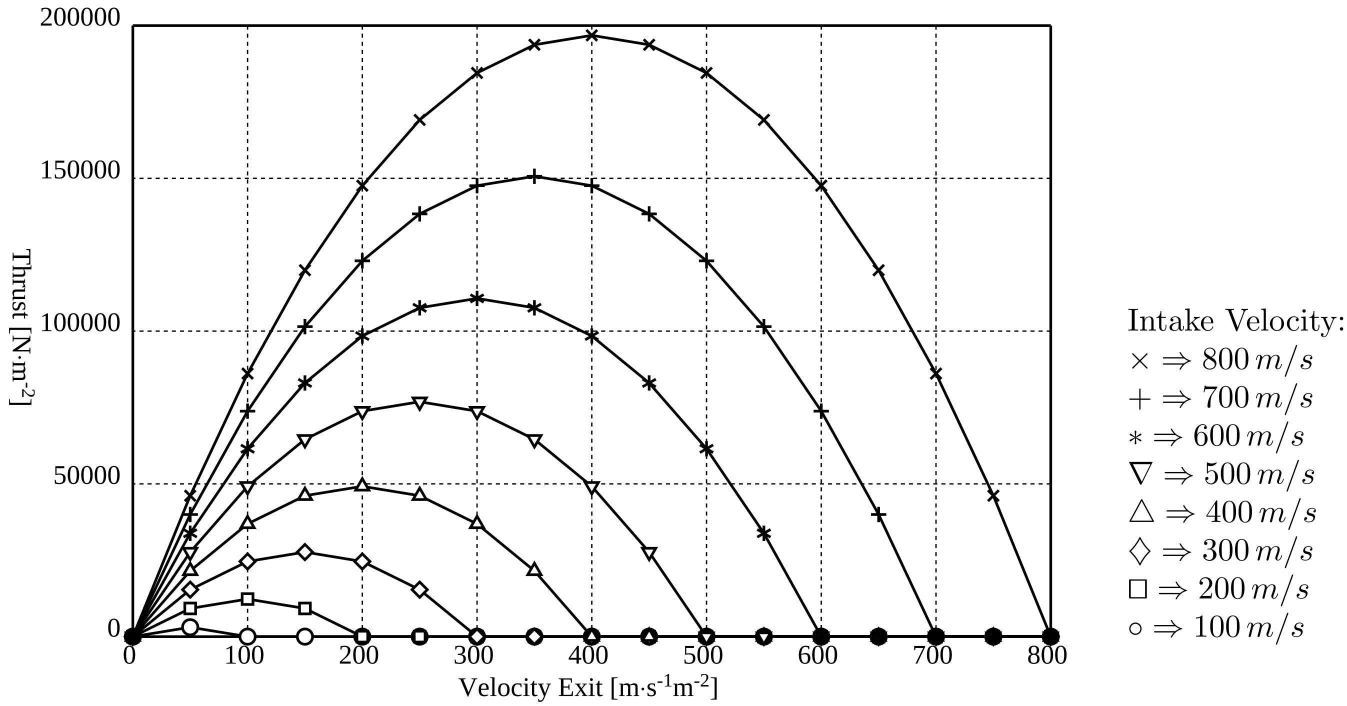

Figure 2. Power per unit area versus exit velocity.

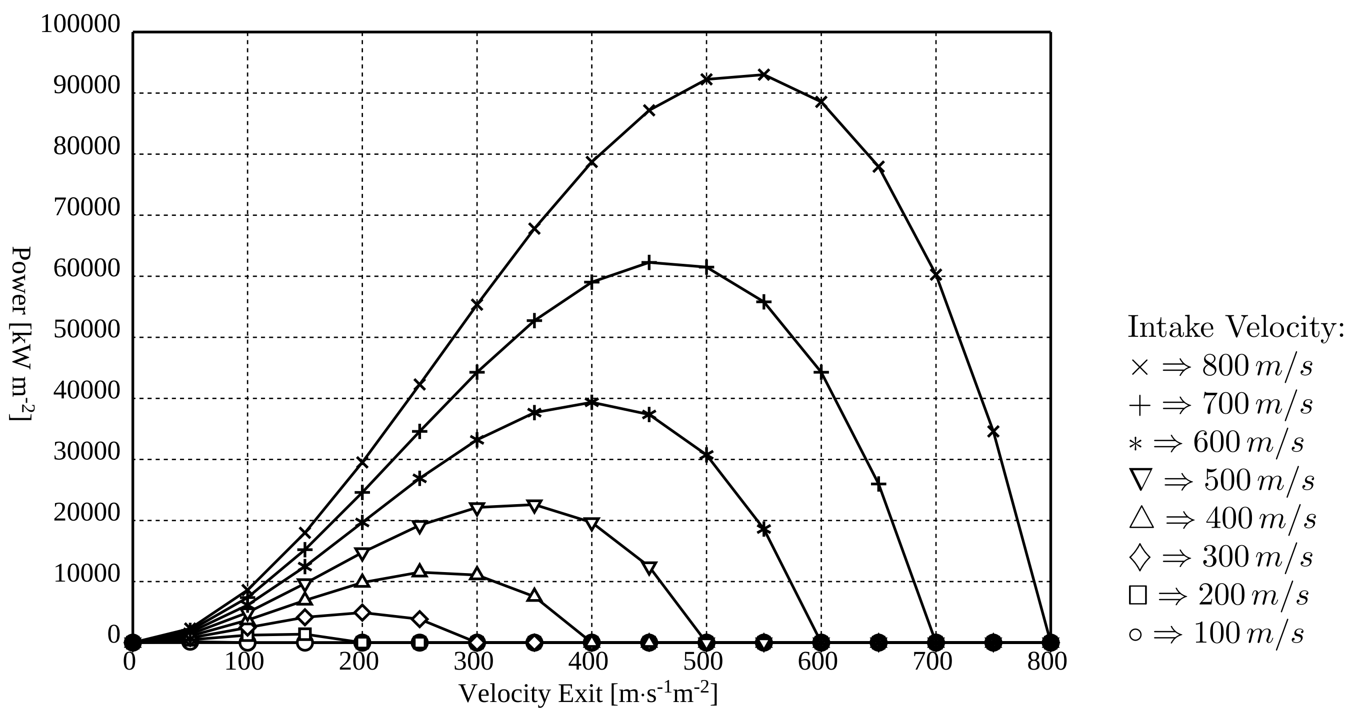

Figure 3. Propulsive efficiency vs Ratio of Exit Velocity to Inlet Velocity. Plot of $\eta_p = \frac{2V_{in}}{(V_{in}+V_{out})}$ where $V_{in} = 1$.