

RLC and LOW PASS FILTER CIRCUITS

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1. RLC circuit simulation by LabView.

The step response of a series RLC circuits was simulated using LabView. A damping ratio of 10 percent approximates a ratio between the first and second peak amplitude of 2 to 1. The damping ratio, $\zeta = \frac{R}{R_{crit}}$ was set equal to 0.1. An Inductance and Capacitance were arbitrary set to 0.1 Farad and 0.1 Henry. Critical Resistance, $R_{crit} = 2\sqrt{\frac{L}{C}}$, was substituted into the damping ratio. Thus,

$$R = \zeta \cdot 2\sqrt{\frac{L}{C}} = 0.1 \cdot 2\sqrt{\frac{0.1}{0.1}} = 0.2 \text{ Ohm}$$

From Plot 1, the cursors were set to measure the time of the peaks. Between the first and second peak, the period was 0.63 seconds yielding a damped natural frequency of $\frac{1}{0.63} = 1.587Hz$. From theory, the natural frequency is

$$f_n = \frac{1}{2\pi} \sqrt{\frac{1}{L \cdot C}} = \frac{1}{2\pi} \sqrt{\frac{1}{0.1 \cdot 0.1}} = 1.59Hz$$

The damped natural frequency is

$$f_d = f_n \cdot \sqrt{1 - \zeta^2} = 1.59\sqrt{1 - .1^2} = 1.58Hz$$

From Plot 1, the ratio of the amplitudes between peak 1 and peak 2 is $\frac{0.86}{0.43=2.0}$. Zeta is approximately equal to $\frac{\ln(2)}{2\pi} = 0.110$. Next, the resistor was changed to more than twice

the 10 percent damping resistance value. With $R=0.41$ Ohm, ζ equals 0.205 and the damped natural frequency is 1.56 Hz. From Plot 2, the time difference between peaks is 0.64 seconds which yields a damped natural frequency of 1.563 Hz. Because the damped natural frequency is $\sqrt{1 - \zeta^2}$ times the natural frequency, the damped frequency changed little. The theoretical amplitude ratio between the peaks is $e^{2 \cdot \pi \zeta} = e^{2 \cdot \pi \cdot 0.205} = 3.626$. From Plot 2, the ratio is $\frac{0.80}{0.2} = 4$. Perhaps this difference is caused by LabView taking steps to solve the differential equations governing the circuit. The plot changes amplitude rapidly in the first few tens of a second and the steps are visible in the plot.

2. Response of LCR circuit to square wave excitation.

The LCR circuit is a second order system. The system was set up as a series connection between a 0.5 H inductor, a 0.1 MFD capacitor and a combined resistance of 5.43 kOhm from the signal generator and inductor. A response plot to a 10 Hz square wave is given in Plot 3. A difference of 1.44 ms was observed between the peaks which yields a damped natural frequency of 694 Hz. From theory, the critical resistance, R_{crit} , equals $2 \cdot \sqrt{\frac{L}{C}} = 4472 \text{ Ohm}$. Since the damping ratio is $\frac{R}{R_{crit}}$, theory indicates a damping ratio of $\frac{495}{4472} = 0.111$. The natural frequency is $\frac{1}{2 \cdot \pi} \sqrt{\frac{1}{L \cdot C}} = 711.8 \text{ Hz}$ and the damped natural frequency is $f_d = f_n \cdot \sqrt{1 - \zeta^2} = 707 \text{ Hz}$. The measured amplitudes were: $e1 = 2.938 \text{ V}$ and $e2 = 1.125 \text{ V}$

so that $\frac{e_1}{e_2} = 2.612$. This results in an approximate damping ratio of $\frac{\ln(2.6125)}{2\pi} = 0.15$.

Sweeping the function generator from 10 Hz to 1000 Hz tested the frequency response of the LCR circuit (Fig 1). A peak does occur at 700 Hz which is near the damped natural frequency of 694 Hz found in the square wave analysis above and 707 Hz from theory. The shape is similar to figure 5.19 in the text which is indicative of a underdamped second order system.

3. Low Pass Filter.

The Low Pass Filter allows low frequency signals to pass while chopping out the high frequency components. The frequency was swept from 10Hz to 50 KHz and the voltage and phase recorded. From theory, the transfer function for this system is

$$\frac{E_{out}}{E_{in}} = \frac{1}{\sqrt{(\omega CR)^2 + 1}}$$

and the phase is

$$\Theta = \text{ARCTAN}(\omega CR)$$

Theoretical and measured data is plotted in Fig 2 and Fig 3.